Tutorial Quiz 2018

MATH1014 - Mathematics and Applications 2

Tutorial Quiz 4
Calculus and Linear Algebra

Reading time: 1 minute
Writing time: 12 minutes

Student Name: ________________
University ID: ________________

Question and Answer Book

Structure of Book

Number of questions	*Number of questions to be answered*	*Number of marks*
1	1	10

- Students are NOT permitted any calculators or notes during the quiz.
- Students are NOT permitted to colaborate in any form during the quiz. Any signs of collaboration or cheating will result in a nullified score and the course convenor will be informed of any academic misconduct.

Materials supplied
- Question and answer booklet of 5 pages.
- Working space is provided throughout the booklet.

Instructions
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the space provided.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

For each statement, decide whether it is always true (**T**) or sometimes false (**F**) and write your answer clearly next to the letter before the statement. In this question, $\mathbf{u}$ and $\mathbf{v}$ are non-zero vectors in $\mathbb{R}^n$; W is a vector space, and $\mathbf{w}_i$ is a vector in W.

(a) The plane with normal vector $\mathbf{u}$ intersects every line in the direction of $\mathbf{u}$.

Proof. **T**. □

(b) Any 5 polynomials in $\mathbb{P}_2$ span $\mathbb{P}_2$.

Proof. **F**. □

(c) If the matrices A and B have the same reduced row echelon form, then $\text{Nul}(A) = \text{Nul}(B)$.

Proof. **T**. □

(d) If V is isomorphic to $\mathbb{R}^5$, then every basis for V has 5 vectors.

Proof. **T**. □

(e) If $\mathcal{P}$ is a change of coordinate matrix, then $\mathcal{P}$ is invertible.

Proof. **T**. □

Question 2

The following questions are multiple choice. Circle one best answer unless the question specifically allows multiple selections.

(a) The improper integral $\int_{-\infty}^{\infty} x^3 dx$ is

(A) Divergent.

(B) Convergent.

(C) Divergent or Convergent depending how we split it up.

Proof. **A.** □

(b) We are given $f(x) > g(x)$ for all $x \in [1, \infty)$. If $\int_1^\infty g(x)dx$ diverges, what can be said about $\int_1^\infty f(x)dx$?

(A) $\int_1^\infty f(x)dx$ converges.

(B) $\int_1^\infty f(x)dx$ diverges.

(C) $\int_1^\infty f(x)dx$ could diverge or converge, we have no way of knowing.

Proof. **B.** □

(c) For $\sum_{n=1}^\infty a_n$ to exist,

(A) It is necessary that $\lim_{n\to\infty} a_n = 0$.

(B) It is sufficient that $\lim_{n\to\infty} a_n = 0$.

(C) It is necessary and sufficient that $\lim_{n\to\infty} a_n = 0$.

Proof. **A.** □

Question 3

Determine the real number $A \in \mathbb{R}$ such that

$$A = \sum_{k=2}^{\infty} \frac{1}{(k+1)(k-1)}.$$

Proof. Using the method of partial fractions, let A, $B \in \mathbb{R}$, such that

$$\frac{1}{(k+1)(k-1)} = \frac{A}{k+1} + \frac{B}{k-1}.$$

Then

$$A(k-1) + B(k+1) = 1 \implies A = -\frac{1}{2},\ B = \frac{1}{2}.$$

Therefore,

$$\begin{aligned}
\sum_{k=2}^{\infty} \frac{1}{(k+1)(k-1)} &= -\frac{1}{2}\sum_{k=2}^{\infty} \frac{1}{k+1} + \frac{1}{2}\sum_{k=2}^{\infty} \frac{1}{k-1} \\
&= -\frac{1}{2}\sum_{k=2}^{\infty} \frac{1}{k+1} + \frac{1}{2}\sum_{k=0}^{\infty} \frac{1}{k+1} \\
&= -\frac{1}{2}\sum_{k=2}^{\infty} \frac{1}{k+1} + \frac{1}{2}\left(1 + \frac{1}{2} + \sum_{k=2}^{\infty} \frac{1}{k+1}\right) \\
&= \frac{3}{4}.
\end{aligned}$$

□